Immigrant Assimilation in a Bilingual Country*

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Abstract

We consider a bilingual country with immigration, where only agents who share the same language can produce together. Immigrants cannot communicate with each other nor with natives unless they learn to speak one of the two languages of the country. We model immigrants’ choice of language. With an exogenous language composition of natives, immigrants tend to choose the language of the majority. Endogenising the choice of minority natives to become bilingual does not alter the set of production partners of other minority members but reinforces the incentives of the immigrants to learn the majority language. In order to learn the minority language, immigrants would require a larger subsidy the smaller the minority. Using Canadian language data from the 2001 Census at the Census Metropolitan Area (CMA) level, we show that the situations of English and French are asymmetric as (i) the capacity of assimilation of French-speaking majorities is smaller, and (ii) English assimilation is larger in CMAs with larger Francophone minorities.

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1 Introduction

Immigrant assimilation is most frequently a hotly debated issue, and becomes even more so in multilingual countries: if the immigrants assimilate, to which language group will they choose to do so? The language chosen by immigrants is an important issue in bilingual countries for at least two reasons. First, immigration is an important source of demographic growth for the language groups; second, as stressed by sociolinguists (see e.g. Fishman, 1967), bilingualism is seldom purely symmetric, and, in particular, one of the two languages has in general a stronger assimilation power among the immigrants. For the case of Canada, for example, Castonguay (1998) argues that “the strength of English resides in its power to assimilation, whereas that of French lies basically in its far greater resistance to Anglicisation compared to Other languages” (p.40). The legal history of Canada contains some well known examples of legislation aimed at favouring assimilation of immigrants in a particular language. For instance, Bill 101 of Quebec (1977) states that “only children whose father or mother received most of their primary education in English, in Quebec, have access to English schools” (Barbaud, 1998, p. 185)\(^1\). Before this law, according to Barbaud (1998) “the primary level bilingual classes [in Quebec] were an avenue to assimilation to English, because almost 70 per cent of immigrant children were registered in English classes” (p. 182).

We begin by presenting preliminary evidence on the determinants of immigrant assimilation in Canada using data from the 2001 Census at the Census Metropolitan Area (CMA) level. Assimilation to the majority language of the CMA is measured comparing the proportion of individuals who have a knowledge of the majority language with the proportion of individuals who have that language as their mother tongue.

Unsurprisingly, assimilation into the majority official language is increasing in the size of the non official tongue minority, which is mainly constituted of immigrants. More interestingly, we show that the situations of English and French are asymmetric in two respects. First, the capacity of assimilation of French-speaking majorities is smaller. Second, English assimilation is larger in CMAs with larger Francophone minorities, which seems to indicate that part of the native French speakers assimilate into the English speaking majorities.

We consider a bilingual country with immigration, where value comes from bilateral production among agents who speak the same language. Immigrants cannot communicate with each other nor with natives unless they learn to speak one of the two languages of the country. We model the decision of learning the language of the native majority or of the native minority as a non cooperative game in which each immigrant takes into account the

\(^1\)This Quebec law had to be modified after the Constitutional Act of 1982 which introduced the so-called “Canada-clause”: “article 23 expressly stipulates that all Canadian citizens whose mother tongue is French or English, or who have received their primary education in one of these languages, have the right to have all their children educated, at the primary and secondary levels, in the same language” (Barbaud, 1998, p.186).
decision of other immigrants in her choice.

We first consider a benchmark situation in which natives cannot modify their initial language endowment. In that case, immigrants tend to choose the language of the majority, as this allows them to produce with more partners. More precisely, if the native majority constitutes more than half of the total population, all immigrants choose to learn the majority language. If instead the native majority is smaller, the interactions among the assimilation decisions of immigrants become relevant at equilibrium and a bandwagon effect can lead either to all the immigrants learning the minority language or to all of them learning the language of the majority.

We determine the socially optimal choice of the immigrants. When the central planner follows an utilitarian criterion, the optimum is reached when the immigrants learn the majority language. Thus, the decentralised equilibrium fails to attain that outcome only when the native majority is small and the immigrants coordinate in the equilibrium in which they all learn the language of the majority. Instead, a Rawlsian central planner would favour assimilation of all the immigrants into the minority language if the native majority constitutes more than half of the population or would otherwise allocate immigrants so as to equalise the sizes of the two language groups after assimilation. Clearly, the decentralised equilibrium does not satisfy the Rawlsian objective.

Next, we endogenise the language composition of natives by allowing minority members to become bilingual as empirically bilingualism is more prevalent among minorities than among majorities. We show that while the choice of minority natives to become bilingual does not alter the set of production partners of other minority members, it reinforces the incentives of the immigrants to learn the majority language as the bilingual minority members can be reached with the majority language. As a result, with (potentially) bilingual minority members, immigrants choose more often to assimilate into the majority language.

Finally, we show that to get immigrants learning the minority language, a subsidy must compensate them for the decrease in the production opportunities they incur by choosing not to learn the majority language. This subsidy must be larger the smaller the minority.

Our model is related to the growing literature on language adoption, and in particular to Church and King (1993) and Lazear (1999). Church and King (1993) shows in a game-theoretic setting that minorities rather than majorities will tend to become bilingual. Lazear (1999) shows how a slow and balanced immigration results in a more rapid assimilation of immigrants than an immigration coming from one particular group.

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2 Other related papers in this literature are John and Yi (2001), and Ortega and Tangerás (2004). John and Yi (2001) develops a dynamic setting to provide an explanation of the factors behind the decline or development of languages. Ortega and Tangerás (2004) studies the political economy of the choice between a unilingual and a bilingual education system.
2 Empirical evidence

This section provides preliminary evidence on the determinants of immigrant assimilation in Canada using data from the 2001 Census. More precisely, we run a series of cross-section regressions at the Census Metropolitan Area (CMA) level. Canada includes CMAs of which six are predominantly French-speaking (Montreal, Ottawa-Gatineau-Quebec part, Sherbrooke, Quebec, Trois Rivières, and Saguenay) while the rest are mainly English-speaking.3

We use information from questions 13 and 16 in the long questionnaire. Question 16 provides information on the mother tongue (English, French, or other, with multiple answers possible). Question 13 provides information on the spoken knowledge of official language(s) (English, French, both of them, neither of them).5 Let \( j \) denote a CMA, \( l \) be a language belonging to the set of languages \( L \), and \( M \) denote the majority mother tongue in a given CMA. We denote by \( \tilde{N}_{jl} \) the number of individuals in CMA \( j \) declaring to have language \( l \) as their mother tongue. As multiple answers to this question are possible, we need to make a choice on how to count individuals who declare to have more than one mother tongue. Here, we simply choose to count an individual who declares to have language \( l \) and language \( l' \) as mother tongue both in \( \tilde{N}_{jl} \) and in \( \tilde{N}_{jl'} \). Then, the proportion of individuals with the majority mother tongue in CMA \( j \) is computed as:

\[
\tilde{x}_j = \frac{\tilde{N}_{jM}}{\tilde{N}_j}
\]

where \( \tilde{N}_j \) is not the total number of individuals but rather \( \tilde{N}_j = \sum_{l \in L} \tilde{N}_{jl} \) i.e. we count \( n \) times the individuals speaking \( n \) languages. Analogously, and using the same rule for multilingual individuals, we define the proportion

\[
x_j = \frac{N_{jM}}{N_j}
\]

of individuals with knowledge of the majority mother tongue \( M \) in CMA \( j \), with in general \( N_j \neq \tilde{N}_j \). We can then define for each CMA \( j \) a simple ratio of assimilation to the majority language as follows:

\[
a_j = \frac{x_j - \tilde{x}_j}{\tilde{x}_j}
\]


4Question 7 is as follows: “What is the language that this person first learned at home in childhood and still understands? If the person no longer understands the language learned, indicate the second language learned.”

5Question 13 is as follows: “Can this person speak English or French well enough to conduct a conversation?”
Table 1: Assimilation ratios in Canadian Census Metropolitan Areas (2001), in percentage

<table>
<thead>
<tr>
<th>Assimilation</th>
<th>Toronto 51.4</th>
<th>Saskatoon 10</th>
<th>Vancouver 45.7</th>
<th>Regina 6.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbotsford</td>
<td>23.1</td>
<td>Oshawa 6.2</td>
<td>Windsor 22.1</td>
<td>Victoria 5.4</td>
</tr>
<tr>
<td>Hamilton</td>
<td>19.6</td>
<td>Kingston 1</td>
<td>Kitchener 19</td>
<td>Halifax -3.1</td>
</tr>
<tr>
<td>Winnipeg</td>
<td>18.9</td>
<td>St John’s -4</td>
<td>Edmonton 17.5</td>
<td>Saint John -5.6</td>
</tr>
<tr>
<td>Calgary</td>
<td>16.2</td>
<td>Montréal (F) -11.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>London</td>
<td>13</td>
<td>Saguenay (F) -14.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. Catherines-Niagara</td>
<td>13</td>
<td>Trois Rivières (F) -18.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greater Sudbury</td>
<td>11.8</td>
<td>Quebec (F) -22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thunder Bay</td>
<td>11.7</td>
<td>Sherbrooke (F) -23.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ottawa-Gatineau (Ontario part)</td>
<td>11.7</td>
<td>Ottawa-Gatineau (Quebec part) (F) -29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(F): CMA with a French mother tongue majority

this ratio determines how the proportion of individuals having knowledge of the majority mother tongue $M$ in CMA $j$ compares (in relative terms) to the proportion of individuals having that same language as a mother tongue. Table 1 presents the values of this assimilation ratio for the 28 CMAs. Thus, for instance, in Toronto, the proportion of individuals with knowledge of English is by 51.4% higher to the proportion of individuals having English as a mother tongue. It can already be noted that assimilation was systematically lowest (and negative) in French-speaking CMAs. Note that a negative assimilation ratio can be in principle due both to a lower tendency for the individuals with a non-official mother tongue (e.g. non English and non French-speaking immigrants) to learn the majority mother tongue or to a higher prevalence of bilingualism in the knowledge of the language than in mother tongues.

We regress this assimilation ratio on a number of CMA-level variables for the 28 CMAs. The results are reported in Table 2. Column 1 shows that there is a positive relationship between assimilation and the proportion of individuals with a non official mother tongue. As these individuals are mostly immigrants\(^6\), this just says that on average immigration reinforces the majority language in CMAs. This result holds also for the other specifications of the regression. In the first regression, we also include the size of the official language minority, i.e. the proportion of English (French) speakers if the CMA is mainly French (English) speaking. The negative and significant coefficient obtained indicates that the language majority gains less first-language speakers among minority members when these

\(^6\)There are also around 200,000 natives having an aboriginal language as their mother tongue.
Table 2: Assimilation in Canadian Census Metropolitan Areas (2001)

<table>
<thead>
<tr>
<th>Dependent variable: Assimilation</th>
<th>1.52***</th>
<th>1.46***</th>
<th>1.46***</th>
<th>1.40***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non official mother tongue minority</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Official mother tongue minority</td>
<td>-2.08***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>French mother tongue minority</td>
<td>2.35***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Majority in mother tongue official language speakers</td>
<td>.20</td>
<td>-3.61***</td>
<td>-3.28***</td>
<td></td>
</tr>
<tr>
<td>( Majority in mother tongue official language speakers )^2</td>
<td>2.43***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>French mother tongue majority</td>
<td>-.21***</td>
<td>-.24***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMA with French mother tongue majority (dummy)</td>
<td>-2.22***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The figures reported are the coefficients obtained from OLS estimation. Standard errors in parentheses. *, ** and *** denote significance at 10%, at 5%, and 1% levels, respectively. Data are from the 2001 Canadian Census, http://www.statcan.ca/english/census01

individuals speak an official language. In other terms, the official status of a minority language is a factor behind the maintenance of the language minority. This may be due for example to regulations such as the Constitutional Act of 1982 which stipulates (art. 23) that all Canadian citizens whose mother tongue is French or English have the right to have all their children educated at the primary and secondary level in the same language (see e.g. Barbaud, 1998). However, this result does not hold symmetrically for both language minorities. Indeed, when interacting the size of the official mother tongue minority with a dummy indicating that the CMA has an English-speaking majority (“French mother tongue minority”), a significant coefficient of the opposite sign is obtained. Thus in English-speaking CMAs with larger French-speaking minorities, assimilation is likely to be more important.

In column 2, we include the proportion of majority members among mother tongue official speakers in order to study whether assimilation to the majority mother tongue is more likely the larger the majority. This is not the case in this regression, but the size of the majority becomes significant in columns 3 and 4 when we introduce also its square as a regressor. More precisely, we get that assimilation depends negatively on the size of the majority and positively on the size of the majority squared. From the estimated values, assimilation is decreasing in the size of the mother tongue majority when the majority corresponds to less than 3/4 of the official mother tongue population and becomes increasing when it surpasses that threshold. However, the regression presented in column 3 shows that there is an additional asymmetry between CMAs in which French is the majority language and

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7The same type of non-linear relation is obtained if the majority is measured as a share of the total speakers instead of as a share of the mother tongue official speakers.
CMAs in which the majority is English-speaking. Indeed, when the proportion of majority members is interacted with a dummy indicating a French-speaking majority in the CMA (“French mother tongue majority”), a negative and significant coefficient is obtained, thus implying that the assimilation power of larger French-speaking majorities is smaller than that of larger English-speaking majorities. The existence of an asymmetry between English and French is also confirmed by the regression in Column 4: assimilation by French-speaking majorities is \textit{ceteris paribus} smaller as the dummy associated to a French-speaking CMA has a negative impact on assimilation.

3 The Model

Consider a country inhabited by a continuum of individuals normalised to 1. The population is constituted of natives $N$ and immigrants $I$. A natives speak the language $a$ while $B$ natives speak language $b$. Initially, each native speaks one language only. The immigrants do not initially speak neither of the native languages and there is a continuum of different immigrant languages. In addition, we assume that immigrants are a minority of the total population, i.e., $I < A + B$.

Value is created through bilateral production between individuals.\footnote{As e.g. in Diamond (1982) or Lazear (1999).} Each individual has the opportunity of producing once with every other individual.\footnote{Equivalently, agents consider their expected payoffs when taking decisions. We are assuming away the possibility that agents belonging to a certain language group are concentrated in a particular location. For an analysis including this geographical dimension, see John and Yi (2001).} Bilateral production occurs if and only if the two partners are able to communicate. We assume that communication is possible only if the two agents speak a common language. If they cannot communicate, the value of production is equal to zero.

In this context, each immigrant can produce only if she learns some native language. Immigrants will thus choose between learning one of the two languages.\footnote{We assume that immigrants cannot become bilingual in the two native languages.} We model this decision as a non cooperative game in which each immigrant takes into account the decision of other immigrants in her choice. We first consider the case in which natives cannot modify their initial language endowment.

3.1 Exogenous language composition of natives

3.1.1 Decentralised equilibrium

Let $\alpha$ denote the proportion of immigrants learning language $a$. Denote by $c$ the cost of learning a native language, assumed to be the same for both languages. The expected utility
$U^a_I$ associated to learning language $a$ for an immigrant is:

$$U^a_I(\alpha) = -c + A + \alpha I$$  \hfill (1)

the immigrant pays cost $c$, ends up speaking language $a$ and can thus produce an amount equal to one with each of the $A$ native $a$ speakers and the $\alpha I$ immigrants who have learned language $a$. Clearly, the payoff associated to learning $a$ is increasing in the number of immigrants who learn $a$.

Analogously, the pay-off associated to learning language $b$ is:

$$U^b_I(\alpha) = -c + B + (1 - \alpha).I$$  \hfill (2)

Subtracting (2) from (1) gives the net benefit $\Delta U^a_I(\alpha)$ for an immigrant of learning language $a$ rather than language $b$:

$$\Delta U^a_I(\alpha) = A - B + (2\alpha - 1).I$$  \hfill (3)

We compute the different Nash equilibria of this game, depending on the exogenous parameters $A$ and $B$. The equilibrium choices of immigrants are presented in Proposition 1.

**Proposition 1** If the native language majority constitutes more than half of the total population, all the immigrants choose to learn the majority language. Otherwise, the assimilation decision is characterised by the existence of multiple equilibria with $\alpha = \{0, \frac{1-2A}{2I}, 1\}$.

**Proof.** Let $\alpha^e$ denote the equilibrium proportion of immigrants learning language $a$. From (3), $\Delta U^a_I(\alpha)$ is increasing in $\alpha$. $\Delta U^a_I(1) = 1 - 2B$ and thus $\alpha^e = 0$ if $B > 1/2$. Symmetrically, $\alpha^e = 1$ if $A > 1/2$. Finally, if $A < 1/2$ and $B < 1/2$, three equilibria co-exist. $\alpha^e = 0$ is an equilibrium since $\Delta U^a_I(0) = 2A - 1 < 0$, $\alpha^e = 1$ is an equilibrium since $\Delta U^a_I(1) = 1 - 2B > 0$ and there is also an (unstable) interior equilibrium $\alpha^e = \frac{1-2A}{2I}$ as $\Delta U^a_I \left(\frac{1-2A}{2I}\right) = 0$. ■

The different equilibria are represented in figure 1 in the $(A, B)$ space for an arbitrarily fixed $I$. Given $I$, the downward sloping line corresponds to the different possible language compositions of the native population.

If the native language majority $L$ constitutes more than half of the total population ($L > 1/2$), all immigrants choose to learn language $L$. Indeed, even in the case in which all immigrants chose to speak the minority language, the number of traders that could be reached by an immigrant learning the minority language $(1-L)$ would be smaller than if she learned the majority language $(L)$. So in that case, the possible interactions in the choices made by different immigrants do not play a role in equilibrium.
In contrast, if the native majority is not too big (i.e. \( A < \frac{1}{2} \) and \( B < \frac{1}{2} \)), the optimal choice for an immigrant depends on the decisions of others immigrants, which leads to multiple Nash equilibria. In the two stable equilibria, all immigrants choose to learn the same language. As a result, after the assimilation decision of immigrants, a majority of the population ends up speaking the native majority language, i.e., the size of the language majority increases with respect to the language minority. Another possible (unstable) equilibrium is one in which a proportion of the immigrants learns the native majority language, while the rest learns the native minority language. As the cost of learning the two languages has been assumed to be the same, an immigrant will be indifferent between learning one language or the other if the total number of speakers is the same for each language at equilibrium. Thus, immigrant assimilation leads in this case to a situation of perfectly balanced bilingualism.

To summarise, the economic incentives for assimilation are such that immigrants tend to choose to learn the language of the native majority group, hence reinforcing the predominance of the majority language among natives.

In order to compare the results of this section with the case of an endogenous language composition of the natives, Fig. 2 represents the results in the \((A, I)\) space:
3.1.2 Welfare

We have seen previously how unpredictable is the language choice of immigrants in a balanced situation and how immigrants choose to join the majority when it exists. We now identify the welfare maximizing language acquisition of immigrants.

This welfare obviously depends on which objective the society has. We consider two different specification of the welfare function. First, if society wishes to maximize the expected number of trade, then we consider a utilitarian specification. Then, if the society wishes to preserve the welfare of the minority group, we could model that preference through a Rawlsian welfare function.

**Utilitarian Welfare** With a utilitarian welfare function, the social planner solves:\(^\text{11}\)

\[
\max_{\alpha} W(\alpha) = AU_A^a(\alpha) + BU_B^b(\alpha) + \alpha I.\bar{U}_I^a(\alpha) + (1 - \alpha)I.\bar{U}_I^b(\alpha)
\]

where \(U_A^a(\alpha)\) and \(U_B^b(\alpha)\) are respectively given by (1) and (2), and

\[
U_A^a(\alpha) = A + \alpha I \\
U_B^b(\alpha) = B + (1 - \alpha)I
\]

\(^{11}\)Here, the welfare of the immigrants is taken into account. This may not be the case if immigrants do not have political rights.
Simplifying (4) using (5) and (6),

$$\max_{\alpha} W(\alpha) = (A + \alpha I)^2 + (B + (1 - \alpha)I)^2 - cI.$$  \hfill (7)

We can then state the following proposition:

**Proposition 2** With utilitarian welfare, immigrants should learn the language of the majority.

**Proof.** $$\frac{\partial W}{\partial \alpha} = 2I(2A + 2\alpha I - 1).$$ If $$A > 1/2,$$ then $$\frac{\partial W}{\partial \alpha} > 0 \ \forall \alpha,$$ i.e. the welfare function monotonically increases in $$\alpha.$$ Then, if $$A > 1/2,$$ the welfare maximising $$\alpha$$ is $$\alpha^* = 1.$$ Symmetrically, $$\frac{\partial W}{\partial \alpha}$$ can be rewritten $$\frac{\partial W}{\partial \alpha} = 2I(1 - 2B - 2I(1 - \alpha))$$ and thus $$\alpha^* = 0$$ if $$B > 1/2.$$ If instead $$A < 1/2$$ and $$B < 1/2,$$ $$\frac{\partial W}{\partial \alpha} = 0$$ for $$\alpha = \frac{1 - 2A}{2}.$$ As $$\frac{\partial W}{\partial \alpha} < 0 \iff \alpha < \bar{\alpha},$$ where $$\bar{\alpha}$$ corresponds to a minimum. Then $$\alpha^* = 0$$ if $$W(0) > W(1) \iff B > A$$ and $$\alpha^* = 1$$ if $$W(0) < W(1) \iff B < A.$$ In the case where $$A = B,$$ both $$\alpha^* = 0$$ and $$\alpha^* = 1.$$ ■

The intuition associated to this proposition is simple. Given that the cost of learning each of the two languages is the same, the utilitarian central planner decides that the immigrants learns the language which opens more production opportunities, i.e. the language of the majority. Thus, optimal immigrant assimilation increases the utility of the native majority while it does not affect the utility of minority members.

Another way of interpreting this result is to study the efficiency costs of bilingualism. In a unilingual country all agents can produce with each other, so social welfare is equal to $$W^{uni} = 1 - cI.$$ Instead, in a bilingual country, $$W(\alpha) = (A + \alpha I)^2 + (B + (1 - \alpha)I)^2 - cI.$$ The cost is thus: $$W^{uni} - W(\alpha) = 1 - (A + \alpha I)^2 + (B + (1 - \alpha)I)^2.$$ This cost is strictly positive since we have assumed in this section that natives cannot shift language and it is minimised when immigrants learn the majority language.

Comparing the decentralised equilibrium with the utilitarian welfare, we can see that the decentralised outcome is efficient whenever the language majority among natives is also a majority in the population as a whole. Instead, if $$A < 1/2$$ and $$B < 1/2,$$ immigrants may coordinate in an inefficient equilibrium in which they learn the language of the minority instead of that of the majority.

### 3.1.3 Rawlsian Welfare

As we have seen, with a utilitarian welfare function, bilingualism is not efficient and everything that can increase the size of the majority improves economic efficiency because it implies more people being able to produce with each other. A society which desires to preserve bilingualism must be assumed to have a different objective. Let us instead assume that the central planner has Rawlsian preferences and thus tries to maximise the utility of the
least favoured agents\textsuperscript{12}. Using (5) and (6), this is given by:

\[ \max_{\alpha} W^R(\alpha) = \min \left( A + \alpha I , \ B + (1 - \alpha) I \right) \]

The maximisation of this function is characterised in the following proposition:

**Proposition 3** With a Rawlsian welfare function, immigrants should choose to learn the language of the minority whenever the native majority constitutes more than half of the total population. Otherwise, immigrants should divide themselves among the two language groups so as to equalize the final sizes of the two language groups.

**Proof.** Denote by \( \alpha^R \) the Rawlsian allocation of immigrants. If \( A > 1/2 \), then the \( b \)-speakers will always be worse-off than the \( a \)-speakers, and thus \( \alpha^R = 0 \). Symmetrically, if \( B > 1/2 \), \( \alpha^R = 1 \). Finally, if \( A < 1/2 \) and \( B < 1/2 \), the function is maximised when \( A + \alpha I = B + (1 - \alpha) I \) and thus \( \alpha^R = \frac{1 - 2A}{2I} \).

In the Rawlsian case, the central planner thus chooses to maximise the production opportunities of the native minority, which leads to the maintenance of bilingualism. Figure 3 represents the Rawlsian optimum:

![Fig. 3 Rawlsian allocation of immigrants across languages](image)

Comparing Figures 1 and 3, it is easy to see that economic incentives push the decentralised

\textsuperscript{12}As \( U_I = \alpha(A + \alpha I) + (1 - \alpha)(B + (1 - \alpha) I) = \alpha U_A^\ast(\alpha) + (1 - \alpha) U_B^\ast(\alpha) \), the expected utility of the immigrants is a convex combination of the utility of the \( a \)-speakers and of the \( b \)-speakers, and thus cannot be lower than the utility of the less favoured of these two groups.
equilibrium in the opposite direction to what would be chosen by a Rawlsian planner. More precisely, if group $L$ constitutes more than half of the population, the decentralised equilibrium is such that immigrants learn the majority language $l$, while the Rawlsian planner would instead choose to allocate them to the minority language. When $A < 1/2$ and $B < 1/2$, the decentralised equilibrium is unlikely to reproduce the Rawlsian optimum as there are multiple equilibria and the interior equilibrium is unstable.

3.2 Bilingual minority members

In this section, we endogenise the language composition of natives by allowing minority members to become bilingual. Empirically, bilingualism is higher among minorities than among majorities. For instance, in Canada, only 9% of the Anglophones consider themselves bilingual whereas 43.4% of the Francophones claim speaking also English. Also, from a theoretical point of view, Church and King (1993) have shown in a game-theoretic framework that the economic incentives for becoming bilingual are stronger among the minority than among the majority, since a minority member learning the majority language gains more potential production partners than a majority member learning the minority language.

Assume that language $a$ is majoritarian among the natives, i.e. $A > B$. We denote by $\beta$ the proportion of minority members who choose to become bilingual.

3.2.1 Decentralised equilibrium without transfers

**Majority language acquisition by minority members** Let $U_B^{ab}$ denote the utility of a minority members who chooses to learn language $a$ and who thus becomes bilingual. This individual pays the cost $c$ of learning the language and can produce with everyone, i.e.,

$$U_B^{ab} = -c + 1$$

(8)

If the individual instead decides to remain unilingual, her utility is still given by (6) i.e. $U_B^b(\alpha) = B + (1 - \alpha)I$. Subtracting (6) from (8), the net gain associated to becoming bilingual is:

$$\Delta U_B^{ab}(\alpha) = -c + A + \alpha I$$

(9)

i.e. the additional partners that can be reached after learning language $a$ (the $A$ natives and the $\alpha I$ immigrants who have learned language $a$) minus the cost.

**Immigrant Language Acquisition** Equations (1), (2) and (3) need to be rewritten to take into account the language choices of the minority members. The utility of an immigrant who decides to learn language $a$ is now:

$$U_I^a(\alpha) = -c + A + \alpha I + \beta B$$

(10)
i.e. she pays learning cost $c$ and can trade as in the benchmark with the native majority members $A$ and the immigrants who learn language $a$, and, in addition, with the $\beta B$ natives that have become bilingual.

$$U^b_I(\alpha) = -c + B + (1 - \alpha)I$$  \hspace{1cm} (11)$$

Subtracting (11) from (10) and simplifying, we obtain the net gain for an immigrant to learn language $a$ instead of language $b$:

$$\Delta U^a_I(\alpha, \beta) = A - B + \beta B + (2\alpha - 1)I$$  \hspace{1cm} (12)$$

Remember that from (3) the net gain of learning language $a$ instead of language $b$ for an immigrant was in the benchmark $\Delta U^a_I(\alpha) = A - B + (2\alpha - 1)I$. The new term in (12) is $\beta B$, which shows that the incentives to learning language $a$ are increasing in the number of minority members that become bilingual.

**Equilibrium** The equilibrium values $(\alpha, \beta)$ are represented in the space $(A, I)$ in the two figures below. Figure 4 studies the case where $c < A$, while in Figure 5, $A < c < A + I$.(for the derivation of the equilibria, see Appendix 1).

In Figure 4, as $c < A$, it is always optimal from (9) to minority members to become bilingual, i.e. $\beta = 1$. As a result, the equilibrium incentives for immigrants to learn language $a$ become greater. Indeed, when comparing Figure 4 with Figure 3, the area in which $\alpha = 1$ is the unique equilibrium is extended to the area with $A > I$ and $A < 1/2$. In addition, $\alpha = 1$ becomes one of the possible equilibria for $A + I < 1/2$. In other words, as the minority members become bilingual, immigrants may all choose to learn the majority language even if the majority is relatively small and/or the number of immigrants itself is small.
In Figure 5, we assume that becoming bilingual for minority members is more expensive than in Figure 4, i.e. $A < c < A + I$. As a result, $\beta < 1$ in some areas of the parameter space. Still, in Figure 5, we can have $\alpha = 1$ as one of the equilibrium outcomes for $A + I < 1/2$, while in this area the immigrants never chose to learn language $a$ in Figure 2 with an exogenous composition of the native population.
3.2.2 Decentralised equilibrium with subsidies and a heterogeneous minority

Assume now that the government can establish a subsidy $s$ for the immigrants choosing to learn the minority language. Assume also the minority members are heterogeneous in their cost $c^{ab}$ of becoming bilingual, with $c^{ab}$ following a uniform distribution in $[0, 1]$. Then, the payoff of remaining unilingual, being bilingual and the net gain of becoming bilingual are still given respectively by (6), (8), and (9), except that now $c^{ab}$ is individual specific.

As for immigrants, the pay-off associated to learning the majority language is still given by (10), but now learning the minority language is subsidised, i.e.,

$$U^b_I(\alpha) = -c + s + B + (1 - \alpha)I$$

and the net benefit of learning the majority language is also correspondingly lower:

$$\Delta U^a_I(\alpha, \beta) = A - B + \beta B + (2\alpha - 1)I - s$$

In order to keep things simple, assume that group $a$ is twice the size of group $b$, i.e. $B = \frac{A}{2}$. Furthermore, as previously, we impose total immigration to be inferior to the previous population, i.e. $I < A + B \iff I < 1/2$. Then, $B > 1/6$ and $A > 1/3$ combining both assumptions.

The characteristics of the equilibrium are summarised in Proposition 4:
Proposition 4 The choice of the minority members to become bilingual reduces incentives for immigrants to learn the minority language. To learn the minority language, immigrants must be given a larger subsidy the smaller the minority.

Proof. To compute $\beta^*$, we need to find the threshold cost $c^{ab}$ corresponding to the marginal $b$-speaker becoming bilingual. This is given by $\Delta U_B^{ab}(\alpha) = -c^{ab} + A + \alpha I = 0$, i.e. $c^{ab} = A + \alpha I$. As $c^{ab}$ is uniform in $[0,1]$, $\beta^* = A + \alpha I$. To solve for the equilibrium choices of immigrants, we use $\beta = \beta^*$ in the expression for $\Delta U^a_I$, i.e., $\Delta U^a_I = 2A + (2B + \alpha I)B + 2\alpha I - 1 - c$. Using $B = \frac{A}{2}$, $\Delta U^a_I(\alpha) = (4 - 5\alpha)B + 2(3\alpha)B^2 + 2\alpha - 1 - c$. As $0 < B < 1/3$, $\Delta U^a_I(\alpha)$ is monotonically increasing in $\alpha$. Since $\Delta U^a_I(1) = 1 - B - B^2 - s$, $\alpha^e = 0$ if $s > 1 - B - B^2$. Analogously, $\Delta U^a_I(0) = 4B + 2B^2 - 1 - s$. Hence, $\alpha^e = 1$ if $s < 4B + 2B^2 - 1$. In the case where $\Delta U^a_I(1) > 0$ and $\Delta U^a_I(0) < 0$ we obtain multiple Nash equilibria: two stable corner solutions where $\alpha^e$ can be 0 or 1 and one interior solution, where the immigrants are indifferent between choosing to learn language $a$ or language $b$. In that case, $\alpha^e = \tilde{\alpha} = \frac{1 + s - 4B - 2B^2}{2 - 3B - 3B^2}$.

Figure 6 represents the different equilibria of the language learned by immigrants depending on the size of the subsidy and of the immigrant population.

In region (I), i.e. for a relatively small number of immigrants and with a sufficiently small subsidy, all the immigrants choose to learn the language of the majority ($\alpha = 1$). For a given size of the minority group, $\alpha = 0$ cannot be reached as an equilibrium unless the subsidy
becomes bigger (i.e. we move to the north). Note also that as the minority becomes larger (the number of immigrants becomes smaller), a larger number of minority members become bilingual\textsuperscript{13}, and this increases the net incentives of immigrants to learn the majority language. Then, the subsidy necessary to create incentives for immigrants to learn the minority language becomes higher, which explains why the frontier of region (I) is upward sloping. In other words, while becoming bilingual has a positive effect on the minority members who choose to do so (as it increases their set of production partners), their bilingualism increases incentives for immigrants to learn the majority language, which in turn affects the utility of the members of the minority who have stayed unilingual.

To get all immigrants learning the minority language, as in region (III), the subsidy $s$ must compensate immigrants for the decrease in the production opportunities they incur by choosing not to learn the majority language. The frontier of region (III) downward sloping because a larger minority group decreases the size of the required compensation.

Finally, multiple equilibria arise in region (II), i.e. when the number of immigrants is high with respect to the number of natives. With a large number of immigrants, the strategic interactions in their language choices become important, and both an equilibrium in which all immigrants learn the majority language and an equilibrium in which all immigrants learn the minority language arise.

### 4 Conclusion

(to be added)

### References


\textsuperscript{13} Actually, the proportion of minority members that become bilingual decreases, but with our hypothesis concerning the relative size of the minority, this effect is dominated by the effect associated to the larger number of minority members.


4.1 Appendix

(to be added)